RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2015
THIRD YEAR [BATCH 2013-16]
MATHEMATICS [Hons]
Paper : VI

Date : 22/12/2015
Time : 11 am – 3 pm
Full Marks : 100

[Use a separate Answer Book for each group]

Group - A

Answer any six questions :

1. Establish Lagrange's polynomial interpolation formula (without error term). Is this polynomial unique? Give reasons. [4+1]

2. Deduce numerical differentiation formula (both first and second order) from Newton's forward interpolation formula. Hence find the value of first and second derivatives at the starting point. [3+2]

3. a) Find the missing terms in the following table

x	:	0	1	2	3	4	5
y	:	0	–	8	15	–	35

[3]

   b) Prove that $hD \equiv \sinh^{-1}(\mu\delta)$ (symbols have their usual meaning) [2]

4. Deduce Newton-Cotes formula for numerical quadrature. Derive Trapezoidal rule from it. [3+2]

5. Describe Gauss-Seidal iterative method for solving a set of linear equations. Also discuss the convergence of the process. [4+1]

6. Establish the Newton-Raphson method for finding a real root of an equation. Establish the condition of convergence. [4+1]

7. Describe the power method to calculate numerically the greatest eigen value of a real square matrix of order n. [5]

8. Find y(4·4) correct to 6 significant figures by modified Euler's method taking h = 0·2 from the differential equation $\frac{dy}{dx} = \frac{2-y^2}{5x}$, y = 1 when x = 4. [5]

9. Solve : $\frac{dy}{dx} = x^2 + y^2$, y(0) = 1 by 4th order Runge-Kutta method, from x = 0 to x = 0·2 with step length h = 0·1. [5]

Answer any four questions :

10. Find the constants a, b, c so that the vector $\vec{F} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ is irrotational and then assuming $\vec{F} = \text{grad}\,\phi$, obtain ϕ. [2+3]

11. a) Find the angle between the surface $x^2+y^2+z^2=9$ and $z = x^2+y^2-3$ at (2, –1, 2). [2]

    b) Find a simplified form of $\vec{\nabla} \times f(r)\vec{r}$; $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. [3]

12. a) A particle moves along the curve $x = t^3+1, y = t^2, z = 5t^2+1$. Find the acceleration at time t = 1 along the vector $\hat{i} + \hat{j} + 3\hat{k}$. [2]

    b) Show that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x,y,z)$ = constant. [3]

13. State Green's theorem in a plane in vector form. Show that the area bounded by a simple closed curve 'C' is given by $\frac{1}{2}\oint_C (xdy - ydx)$. [2+3]

14. Evaluate by Stoke's theorem $\oint_\Gamma (\sin z\, dx - \cos x\, dy + \sin y\, dz)$ where Γ is the boundary of the rectangle $0 \le x \le \pi, 0 \le y \le 1, z = 3$. [5]

15. State Gauss Divergence theorem and use it to prove that $\iiint_V \vec{\nabla}\phi\, dv = \iint_S \phi \hat{n}\, dS$, where $\phi(x,y,z)$ is a scalar function and $\hat{n}$ is the outward drawn unit normal vector to the surface S. [2+3]

Group - B

Answer **any three** questions :

16. a) Prove that the time of a small oscillation of a compound pendulum is minimum when the axis of suspension is parallel to the maximum radius vector of the momental ellipsoid at the centre of inertia and the point of suspension is so taken that the centre of inertia bisects the join of the point of suspension and the centre of oscillation. [7]

    b) An elliptic lamina is such that when it swings about one latus rectum as a horizontal axis, the other latus rectum passes through the centre of oscillation. Find the eccentricity of the ellipse. [3]

17. a) Prove that the resultant kinetic energy of a rigid body moving in two dimensions under finite forces is equal to the sum of two kinetic energies, one due to translation and the other due to rotation. [6]

    b) A wire is in the form of a semi-circle of radius 'a'. Show that at an end of its diameter, the principal axes in its plane are inclined to the diameter at angles $\frac{1}{2}\tan^{-1}\frac{4}{\pi}$ and $\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\frac{4}{\pi}$. [4]

18. a) Establish the Principle of Conservation of Angular Momentum under finite forces. [4]

    b) A thin rod of length '2a' revolves with uniform angular velocity ω about a vertical axis through a small joint at one extremity of the rod, so that it describes a cone of semi-vertical angle α. Show that $\omega^2 = \frac{3g}{4a\cos\alpha}$. [6]

19. a) A uniform rod is held at an inclination α to the horizontal with one end in contact with the horizontal table whose coefficient of friction is μ. If it be then released, then show that it will commence to slide if $\mu < \frac{3\sin\alpha\cos\alpha}{1+3\sin^2\alpha}$. [6]

    b) If A, B, C; D, E, F be the moments and products of inertia of a rigid body about a given set of rectangular axes OX, OY, OZ, then find the moment of inertia of the body about a line having direction cosines ℓ, m, n with respect to the given set of axes. [4]

20. a) Two equal uniform rods AB and AC are freely jointed at A. They are placed on a table so as to be at right angles. The rod AC is struck by a blow at C in a direction perpendicular to AC. Show that the resulting velocities of the middle points of AB and AC are in the ratio 2:7. [5]

    b) An elliptic area of eccentricity e, is rotating with angular velocity ω about one latus rectum. Suddenly this latus rectum is loosed and the other is fixed. Find the new angular velocity. [5]

Answer **any two** questions :

21. a) Find the Escape Velocity at an altitude of 900 km above the surface of the earth.
    Given, Radius of the Earth $R_e = 64\times10^7$ cm , Mass of the Earth $M_e = 6\times10^{27}$ gms. Universal gravitational constant 'G' = $6\cdot66\times10^{-8}$ cgs unit. [4]

b) When a periodic comet is at its greatest distance from the Sun, its velocity 'v' is increased by a small quantity 'δv'. Show that the comet's least distance from the Sun is increased by the quantity $4\cdot\delta v\cdot\left\{\frac{a^3(1-e)}{\mu(1+e)}\right\}^{1/2}$, the symbols having their usual meanings. [6]

22. a) Prove that the velocity at the end of the minor axis of a planet's orbit is the geometric mean of the velocities when it is at the perihelion and the aphelion respectively. [4]

b) Find the time to describe an arc of a parabolic orbit when a particle moves under Inverse square law. [6]

23. a) The volume of a spherical raindrop falling freely increases at each instant of time by an amount equal to μ times its surface area at that instant. If the initial radius of the raindrop be 'a' then show that it has fallen through a distance $\frac{9a^2g}{32\mu^2}$, when its radius becomes 2a. [7]

b) Write down the Kepler's laws of planetary motion. [3]

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